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MODELING OF EQUATION FOR THE DISSIPATION RATE OF REYNOLDS STRESSES

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All the correlations involved in the equation for the rate of dissipation of kinetic turbulence energy are modeled. Approximations derived earlier are used as model relations. Modeling is performed by direct comparison of the approximations with the correlations obtained as a result of a numerical solution of the Navier–Stokes unsteady-state equations. The form of the approximations and the empirical coefficients are determined. Approximations for correlations involved in the equation for the tensor dissipation function are obtained. A new equation for the rate of turbulence-energy dissipation is proposed. The results of the work can be used to determine the Reynolds stresses in second-order turbulence models.

Introduction. The velocity field in turbulent flow can be described using the Navier–Stokes unsteady-state equations. An exact numerical solution was obtained for flows in which the turbulent Reynolds number was comparatively small [1]. The ratio of the scales of large and small vortices increases with it. To describe small-scale motion with acceptable accuracy, we have to increase the number of points of the grid. Under these conditions, different turbulence models are used for solution of the applied problems of hydrodynamics, since even for moderate turbulent Reynolds numbers, the power of the available computers turns out to be insufficient to solve the Navier–Stokes unsteady-state equations.

Second-order turbulence models are the most popular at present. A method of their construction was proposed in the early seventies [2]. The essence of this approach is that the Reynolds stresses must be determined from exact equations for the second single-point moments

$$\frac{DR_{ij}}{Dt} = F_{ij} + P_{ij} + \Phi_{ij} - 2\varepsilon_{ij} + D_{ij}\,, \tag{1}$$

In Eq. (1), convective transfer and generation processes are described accurately. The term that contains the pulsations of the external force depends on its form. For stratified flows, this term is known. The terms that contain the correlations of pressure pulsations and dissipation and diffusion terms must be modeled, since they contain unknown correlations of a higher order.

In [2], the results of investigations performed before 1974 are generalized and a complete second-order turbulence model is proposed. To model the correlation Φ_{ij}, use was made of the earlier Rott formula. In subsequent works [3, 4], a general form of the approximation for Φ_{ij} was obtained using the method of invariant modeling and the realizability condition. The unknown coefficients involved in the relation for Φ_{ij} were determined based on the data of direct numerical simulation (DNS) in [5, 6]. The approximation of the diffusion term is based on the hypothesis of the gradient nature of turbulent diffusion. To determine ε_{ij}, use was made of the Kolmogorov hypothesis of local isotropy of the velocity field, in accordance with which a tensor dissipation function is determined from the relation $\varepsilon_{ij} = \varepsilon(\delta_{ij}/3)$. However, the data of direct numerical modeling of flow in a boundary layer showed [7] that the assumption of local isotropy of small-scale motion is not fulfilled, at least, for small and moderate turbulent Reynolds numbers. In [8, 9], the Navier–Stokes equations in the Fourier space are analyzed and the following conclusion is drawn: anisotropy of turbulence in large-scale motion induces anisotropy of turbulence in small-scale motion for any Reynolds numbers.

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The first attempts to allow for the anisotropy of dissipation processes in turbulent flows were made only in recent years. In [10], an algebraic relation for ε_{ij} was proposed. An exact differential equation for the tensor dissipation function ε_{ij} that can be derived from the Navier–Stokes equations is used in [11-14]. It contains a number of unknown correlations of a higher order that must be expressed in terms of the known correlations. Only after this can it be used for numerical calculations. To model these correlations, universally accepted methods dating back to the works by Kolmogorov and Rott and new techniques, in particular, the method of invariant modeling, were employed.

The present work seeks to model the unknown correlations in the equation for a tensor dissipation function. The relations of [11-14] are taken as the initial approximations. To determine the unknown coefficients and to select an acceptable form of approximation, we used the data of direct numerical modeling of developed turbulent flow in a channel [1]. Special attention was paid to the approximation of the individual terms of the equation for the dissipation rate in strongly anisotropic turbulence. Unlike in earlier works, the form of approximation and empirical coefficients were determined by direct comparison of the general form of the approximation for the correlation sought with the data of DNS.

1. Basic Equations. The exact equation for the tensor dissipation function has the form

$$\frac{D\varepsilon_{ij}}{Dt} = F_{(\varepsilon)ij} + P_{(\varepsilon 1)ij} + P_{(\varepsilon 2)ij} + P_{(\varepsilon 3)ij} + P_{(\varepsilon 4)ij} + \Pi_{(\varepsilon)ij} + D_{(\varepsilon)ij} + T_{(\varepsilon)ij} - Y_{(\varepsilon)ij}. \tag{2}$$

In Eq. (2), there are five different terms that describe the generation of the tensor of dissipation rate: $F_{(\varepsilon)ij}$, $P_{(\varepsilon 1)ij}$, $P_{(\varepsilon 2)ij}$, $P_{(\varepsilon 3)ij}$, and $P_{(\varepsilon 4)ij}$. The term $\Pi_{(\varepsilon)ij}$ determines the redistribution of ε_{ij} in terms of pressure pulsations, $Y_{(\varepsilon)ij}$ determines the ductile fracture of small-scale vortices, and $D_{(\varepsilon)ij}$ and $T_{(\varepsilon)ij}$ describe the viscous and turbulent diffusions of ε_{ij}, respectively.

Within the framework of the complete second-order turbulence model, the correlations $P_{(\varepsilon 1)ij}$ and $D_{(\varepsilon)ij}$ are expressed in terms of the known correlations and therefore require no modeling. The remaining terms in the right-hand side of Eq. (2) contain the unknown correlations, which must be expressed in terms of the known correlations in order to obtain a closed system of equations. The state of modeling of the written correlations is as follows. In [13], Eq. (2) was modeled for uniform turbulent flow, in which the diffusion terms and the term $P_{(\varepsilon 3)ij}$ are zero. The terms $\Pi_{(\varepsilon)ij}$, $P_{(\varepsilon 4)ij}$, and $Y_{(\varepsilon)ij}$ were not modeled separately but an approximation for the difference of the terms $(P_{(\varepsilon 4)ij} - Y_{(\varepsilon)ij})$ was proposed. Approximations for $\Pi_{(\varepsilon)ij}$, $P_{(\varepsilon 2)ij}$, and $Y_{(\varepsilon)ij}$ are proposed in [14]. The results of the modeling were not compared directly with the data of DNS in the above works.

An exact equation for the scalar dissipation rate can be obtained by convolution of Eq. (2):

$$\frac{D\varepsilon}{Dt} = F_{\varepsilon} + P_{\varepsilon 1} + P_{\varepsilon 2} + P_{\varepsilon 3} + P_{\varepsilon 4} + \Pi_{\varepsilon} + T_{\varepsilon} + D_{\varepsilon} - Y_{\varepsilon}, \tag{3}$$

where all the terms are the result of convolution of the corresponding terms of Eq. (2). In [12], approximations for the terms $P_{\varepsilon 3}$, T_{ε}, and the sum of the terms $(P_{\varepsilon 1} + P_{\varepsilon 2} + P_{\varepsilon 4} - Y_{\varepsilon})$ are proposed. The terms written in parentheses were not modeled separately. The results of the modeling were compared with the data of DNS. According to the results of the comparison, the authors draw the conclusion that the model proposed has advantages over the known equation for the dissipation rate. We note that in this approach to modeling, the probability exists that the model for the sum of the terms $(P_{\varepsilon 1} + P_{\varepsilon 2} + P_{\varepsilon 4} - Y_{\varepsilon})$ could describe rather well the data of DNS for flow in a channel but in doing so be unsuitable for description of other flows.

2. Models for the Unknown Correlations. To model the unknown correlations in Eq. (2), the authors of [11] used the method of tensor approximation. Its essence is that the tensor of the unknown correlations is expanded into a series with allowance for the symmetry of indexes in terms of the basis of the known correlations obtained from the initial tensor by convolution. The character of the relationship between the tensor sought and the basis was assumed to be linear. The tensor equalities were written using tensor coefficients, whose form was determined using the theory of invariants. As a result of modeling, the following relations for the unknown correlations are obtained:

$$\frac{K}{\varepsilon^2} F_{(\varepsilon)ij} = -\frac{\beta\nu}{\nu + a}\frac{K}{\varepsilon^2}\left(g_i\varepsilon_{(\tau)j} + g_j\varepsilon_{(\tau)i}\right), \tag{4}$$

$$\frac{K}{\varepsilon^2} P_{(\varepsilon 1)ij} = -\frac{K}{\varepsilon}\left(\frac{\varepsilon_{ik}}{\varepsilon} U_{j,k} + \frac{\varepsilon_{jk}}{\varepsilon} U_{i,k}\right), \tag{5}$$

$$\frac{K}{\varepsilon^2} P_{(\varepsilon 2)ij} = -a_1\frac{K}{\varepsilon}\left(U_{i,j} + U_{j,i}\right) + a_2\frac{K}{\varepsilon}\left(\frac{\varepsilon_{ik}}{\varepsilon} U_{j,k} + \frac{\varepsilon_{jk}}{\varepsilon} U_{i,k} + \frac{\varepsilon_{ik}}{\varepsilon} U_{k,j} + \frac{\varepsilon_{jk}}{\varepsilon} U_{k,i}\right), \tag{6}$$

$$\frac{K}{\varepsilon^2} P_{(\varepsilon 3)ij} = -\nu\,[(\xi K_{,k} + \xi_2 R_{mk,m})(U_{i,kj} + U_{j,ki}) + (\xi_1 R_{ki,m} - \xi R_{km,i})\,U_{j,km} + (\xi_1 R_{kj,m} - \xi R_{km,j})\,U_{i,km} + (\xi K_{,i} + \xi_2 R_{mi,m})\,U_{j,kk} + (\xi K_{,j} + \xi_2 R_{mj,m})\,U_{i,kk}], \tag{7}$$

$$\frac{K}{\varepsilon^2} P_{(\varepsilon 4)ij} = \gamma_1 + \gamma_2\delta_{ij}, \tag{8}$$

$$\frac{K}{\varepsilon^2} Y_{(\varepsilon)ij} = \gamma_3 d_{ij} + \gamma_4\delta_{ij}, \tag{9}$$

$$\frac{K}{\varepsilon^2} T_{(\varepsilon)ij} = C_{s2}\frac{K}{\varepsilon^2}\left[\frac{K}{\varepsilon}\left(R_{km}\varepsilon_{ij,m} + R_{im}\varepsilon_{jk,m} + R_{jm}\varepsilon_{ik,m}\right)\right]_{,k}, \tag{10}$$

$$\frac{K}{\varepsilon^2}\Pi_{(\varepsilon)ij} = \frac{K}{\varepsilon^2}\left[\alpha_{1\varepsilon}\left(\varepsilon_{ik}U_{j,k} + \varepsilon_{jk}U_{i,k} - \frac{2}{3}\varepsilon_{mn}U_{m,n}\delta_{ij}\right) + \alpha_{2\varepsilon}\left(\varepsilon_{ik}U_{k,j} + \varepsilon_{jk}U_{k,i} - \frac{2}{3}\varepsilon_{nm}U_{n,m}\delta_{ij}\right) + \alpha_{3\varepsilon}\varepsilon\left(U_{i,j} + U_{j,i}\right)\right] + \gamma_5 d_{ij}. \tag{11}$$

The approximations for the terms of Eq. (3) can be obtained by convolution of Eqs. (4)-(11):

$$\frac{K}{\varepsilon^2} P_{\varepsilon 1} = -2\frac{\varepsilon_{ik}}{\varepsilon}\frac{K}{\varepsilon} U_{k,i}, \tag{12}$$

$$P_{\varepsilon 2} = 2a_2 P_{\varepsilon 1}, \tag{13}$$

$$\frac{K}{\varepsilon^2} P_{\varepsilon 3} = -2\nu\frac{K}{\varepsilon^2}\,[(\xi_1 R_{ki,m} - \xi R_{km,i})\,U_{i,km} - (\xi K_{,i} + \xi_2 R_{mi,m})\,U_{i,mm}], \tag{14}$$

$$\frac{K}{\varepsilon^2} P_{\varepsilon 4} = 3\gamma_2, \tag{15}$$

$$\frac{K}{\varepsilon^2} Y_{\varepsilon} = 3\gamma_4, \tag{16}$$

$$\frac{K}{\varepsilon^2} T_\varepsilon = C_{s2} \frac{K}{\varepsilon^2} \left[\frac{K}{\varepsilon} (R_{km} \varepsilon_{,m} + 2R_{im} \varepsilon_{ik,m}) \right]_{,k} , \tag{17}$$

$$\Pi_\varepsilon = 0 . \tag{18}$$

A general form of approximations for unknown correlations is obtained in [11]. The empirical coefficients involved in the approximations were not determined. Approximation relations obtained in [13] have approximately the same form as relations (5)-(18).

Unlike in [11, 13], an equation for double-point correlation of velocity pulsations was written in [14]. Its analysis showed that we can introduce a tensor that characterizes the integral scale. For this tensor, we can write a corresponding differential equation that contains five independent coefficients. In [14], the coefficients of the equation for the scale were determined and the method for transforming it into the equation for the tensor dissipation function was shown. Approximations that are somewhat different from the above were obtained as a result of this investigation [14]. In particular, the approximation for $P_{(\varepsilon 2)ij}$ has the following form:

$$\frac{K}{\varepsilon^2} P_{(\varepsilon 2)ij} = 2.5 \frac{\varepsilon_{ij}}{\varepsilon} \frac{P_k}{\varepsilon} - \frac{1}{15a_0} \left[\left(2 \frac{\varepsilon_{ij}}{\varepsilon} - \frac{1}{3} \delta_{ij} \right) (4 + 14a_1) + 5a_0 \delta_{ij} \right] \frac{K}{\varepsilon^2} P_{\varepsilon 1} , \tag{19}$$

$$\frac{K}{\varepsilon^2} P_{\varepsilon 2} = 2.5 \frac{P_k}{\varepsilon} - \frac{1}{15a_0} [15a_0 + (4 + 14a_1)] \frac{K}{\varepsilon^2} P_{\varepsilon 1} , \tag{20}$$

where

$$a_0 = 0.05 , \quad a_1 = 0 . \tag{21}$$

The approximations proposed are tested indirectly in [13, 14]. For this purpose, the differential equation (1) simultaneously with the corresponding approximations for Φ_{ij} and D_{ij} and Eq. (1) with the approximations written above for the terms of this equation were solved for homogeneous flow with a constant velocity shift for flow in a channel and for some other flows. Next, the results of the calculations were compared with the data of direct numerical modeling of those flows. We note that numerous approximations, each containing several empirical coefficients, are involved in Eqs. (1) and (3). Under these conditions, an incorrect description of one correlation can always be compensated for by the coefficients involved in the approximations for other correlations. Therefore based on this comparison we can infer that the total action of all the above approximations permits calculation of one or another flow with one or another degree of accuracy. It is quite impossible to make a conclusion about the exactness of each approximation written above.

An alternative approach is as follows. To evaluate the exactness of relations (4)-(21), we need to directly compare the data of DNS with the results of calculations by the corresponding approximating relation for each correlation involved in Eqs. (2) or (3). Up to now, direct comparisons of this type have not been performed; therefore, the problem of selecting an approximation out of the approximations written above and of the coefficients involved in these approximations remains to be solved.

Data of DNS for Developed Flow in a Channel. It has been shown in the previous section that, for the same correlation, approximations that differ from each other are proposed in different works. To evaluate the exactness of different approximations, we use the data of direct numerical modeling of developed flow in a channel [1]. Figure 1 shows the distributions of the components of the dissipation tensor along the transverse coordinate. They differ rather strongly from isotropic values, which can be determined from the relation

$$\varepsilon_{ij} = \frac{1}{3} \varepsilon \delta_{ij} . \tag{22}$$

The deviation from isotropy increases as the wall is approached. Figure 2 shows the distributions of correlations involved in the equation for a scalar rate of dissipation. All the quantities are made dimensionless by division of

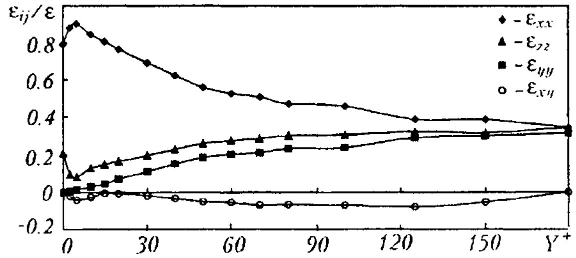


Fig. 1. Distribution of the components of the tensor ε_{ij} in channel.

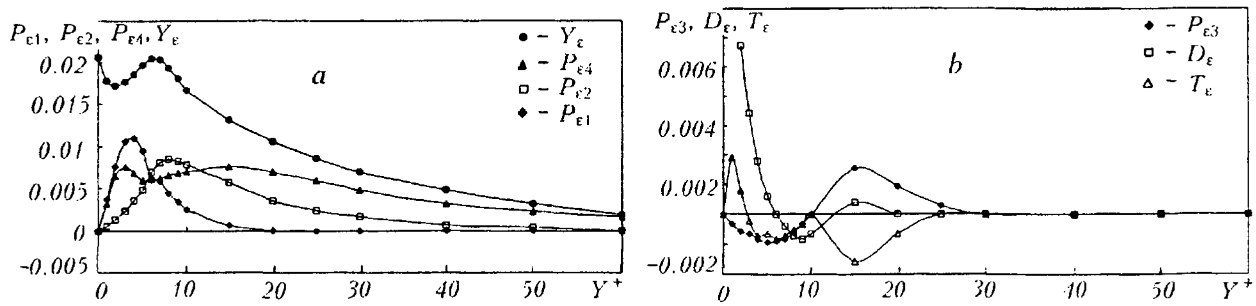


Fig. 2. Data of DNS for correlations in the equation for the rate of dissipation of turbulence kinetic energy.

the corresponding term by U_τ^6/ν^2. From the figures it follows that the terms $P_{\varepsilon4}$ and Y_ε predominate in the equations for the dissipation rate. However, in the vicinity of the wall, the contribution of the terms of generation $P_{\varepsilon1}$ and $P_{\varepsilon2}$ becomes substantial. On the wall itself, the term Y_ε is balanced by viscous diffusion D_ε. The small terms $P_{\varepsilon3}$, T_ε, and D_ε are shown in Fig. 2b. The term Π_ε is negligibly small in the entire region of the flow.

4. Results of Modeling. Data for correlations contained in Eq. (2) are not given in [1]. Therefore, for modeling we can use only the correlations that are involved in (3) for the dissipation rate ε. We obtained some information on the coefficients and the form of the approximations for the correlations $P_{(\varepsilon1)ij}$, $P_{(\varepsilon2)ij}$, $P_{(\varepsilon3)ij}$, $P_{(\varepsilon4)ij}$, $\Pi_{(\varepsilon)ij}$, $Y_{(\varepsilon)ij}$, $T_{(\varepsilon)ij}$, and $D_{(\varepsilon)ij}$ with allowance for the fact that the terms of Eq. (3) are convolutions of the corresponding tensors of Eq. (2).

Correlation $P_{(\varepsilon1)ij}$. Relations (5) and (12) for $P_{(\varepsilon1)ij}$ and $P_{(\varepsilon1)}$ are exact and because of this there is no need to model them. Comparisons of the data of DNS with the results of calculations by formula (12) given in Fig. 3 suggest that the agreement of the data of DNS with calculation is rather good, especially in the wall region of the flow. Away from the wall the effect of this term is small.

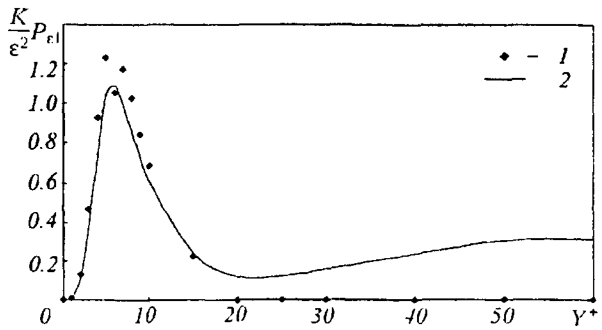
Correlation $P_{(\varepsilon2)ij}$. To model this correlation, the authors of [11, 13, 14] propose to use approximations (6) and (19). The corresponding approximations for the correlation $P_{\varepsilon2}$ have the form of (13) and (20). Figure 4 shows a comparison of the data of DNS with the results of calculations by the formulas proposed. From the figure it can be seen that approximations (13) and (20) describe unsatisfactorily the results of direct numerical modeling. Processing of the data of DNS showed that their best description can be obtained when we use approximation (20) in the form

$$\frac{K}{\varepsilon^2} P_{\varepsilon2} = \frac{P_k}{\varepsilon}. \tag{23}$$

Accordingly, for the approximation of the correlation $P_{(\varepsilon2)ij}$, we can recommend the relation

$$\frac{K}{\varepsilon^2} P_{(\varepsilon2)ij} = \frac{\varepsilon_{ij}}{\varepsilon} \frac{P_k}{\varepsilon}. \tag{24}$$

Correlation $P_{(\varepsilon3)ij}$. The data of Fig. 2 indicate that this correlation is small as compared to other terms of Eq. (3). For it, approximation (14) is proposed in [11] which in a rectangular coordinate system takes the form



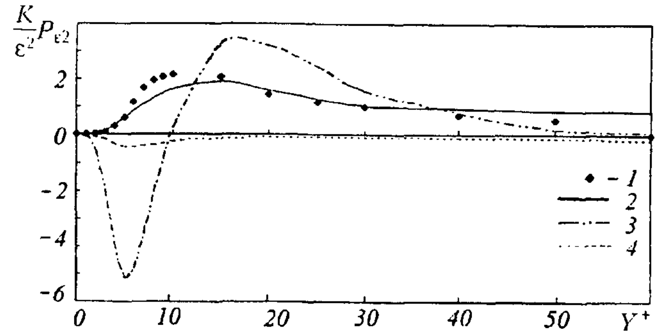


Fig. 3. Comparison of the results of calculations with the data of DNS for the correlation $P_{\varepsilon 1}$: 1) data of DNS; 2) calculation by formula (12).

Fig. 4. Comparison of the results of calculations with the data of DNS for the correlation $P_{\varepsilon 2}$: 1) data of DNS; 2) calculation by formula (23); 3) by formula (20); 4) by the model of [13].

$$\frac{K}{\varepsilon^2} P_{(\varepsilon 3)xy} = -2 (\xi_1 - \xi_2) R_{xx,y} U_{x,yy} . \tag{25}$$

Figure 5 shows a comparison of the data of DNS with calculation by formula (25) for $(\xi_1 - \xi_2) = -0.5$. According to the figure, approximation (25) describes unsatisfactorily the results of direct numerical modeling; therefore, allowing for the small contribution of this correlation to the total balance of the equation of conservation of the dissipation rate, we can assume $P_{(\varepsilon 3)} = 0$ as a first approximation.

Correlation $P_{(\varepsilon 4)ij}$. From the distributions of Fig. 2a it follows that the correlation $P_{\varepsilon 4}$ is the main source term in the equation for the dissipation rate. We note that practically all second-order turbulence models use currently an equation for the dissipation rate in the form

$$\frac{D\varepsilon}{Dt} = C_{\varepsilon 1} \frac{\varepsilon}{K} P_k - C_{\varepsilon 2} \frac{\varepsilon^2}{K} + C_\varepsilon \left(\frac{K}{\varepsilon} R_{ij} \varepsilon_{,j} \right)_{,i} , \tag{26}$$

where $C_{\varepsilon 1} = 1.45$; $C_{\varepsilon 2} = 1.9$; $C_\varepsilon = 0.15$.

Equation (26) is obtained based on intuitive notions that the time variation in the dissipation rate is determined by a source term that is proportional to the generation of turbulence energy, the runoff term, and the diffusion term. In relation (26), it is assumed in implicit form that

$$\frac{K}{\varepsilon^2} P_{\varepsilon 4} = C_{\varepsilon 1} \frac{P_k}{\varepsilon} . \tag{27}$$

Approximations (8) and (15) proposed in [11] differ fundamentally from expression (27). The scalar coefficients γ_1 and γ_2 can, in principle, depend on the scalar invariants $\frac{P_k}{\varepsilon}$, $\frac{K}{\varepsilon^2} P_{\varepsilon 1}$, X_1, II_b, III_b, F_b, or II_d, III_d, and F_d.

Processing of the results of DNS showed that the dependence of the coefficient γ_2 on the above invariants can be sought in the form

$$3\gamma_2 = a_1 \frac{K}{\varepsilon^2} P_{\varepsilon 1} + a_2 \frac{P_k}{\varepsilon} + a_3 X_1 + a_4 F_b . \tag{28}$$

To determine the coefficients a_i, we used the data of Fig. 2. As a result of analyzing, we obtained the following coefficients:

$$a_1 = 1.75 ; \quad a_2 = 3.4 ; \quad a_3 = -0.29 ; \quad a_4 = 3.6 . \tag{29}$$

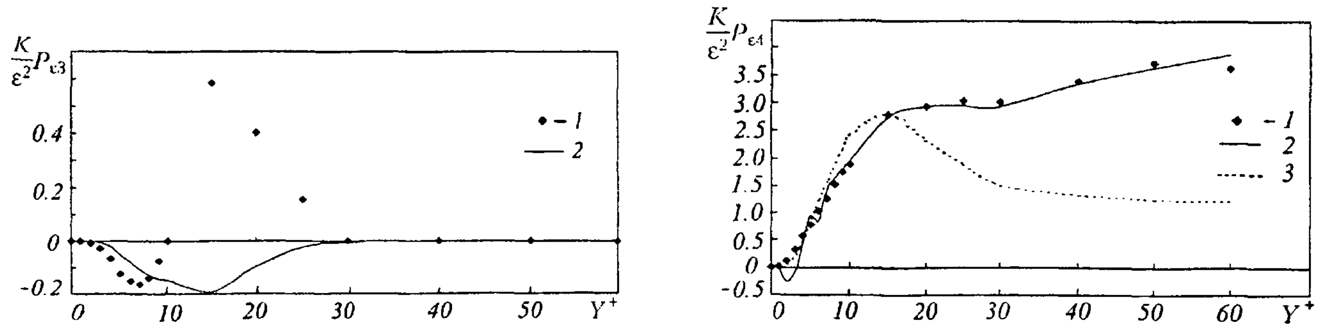


Fig. 5. Comparison of the results of calculations with the data of DNS for the correlation $P_{\varepsilon3}$: 1) data of DNS; 2) calculation by formula (25).

Fig. 6. Comparison of the results of calculations with the data of DNS for the correlation $P_{\varepsilon4}$: 1) data of DNS; 2) calculation by formulas (15) and (28); 3) by formula (28); 2) by formula (27).

Figure 6 shows the distribution of the function $\frac{K}{\varepsilon^2}P_{\varepsilon4}$ across the thickness of the boundary layer, from which it follows that in the entire region of the flow the data of DNS are in good agreement with calculation by formulas (28) and (29). From the figure it can be seen that calculation by formula (27) can cause a large error. Thus, to calculate the correlations considered, we can recommend approximations (8) and (15), whose coefficients are determined by relations (28) and (29). Equation (28) makes it possible to calculate only the coefficient γ_2. For determination of γ_1, additional investigations are required.

Correlation $Y_{(\varepsilon)ij}$ describes the ductile fracture of small-scale vortices. In [11], it is recommended to use approximations (9) and (16) for the correlations $Y_{(\varepsilon)ij}$ and Y_ε. In accordance with Eq. (27) an approximation that coincides completely with relation (16), provided that $3\gamma_4 = C_{\varepsilon 2}$, is proposed in [2] for determination of the correlations Y_ε. The dependence of γ_4 on scalar invariants was sought in the form

$$3\gamma_4 = b_1 \frac{K}{\varepsilon^2} P_{\varepsilon1} + b_2 \frac{P_k}{\varepsilon} + b_3 X_1 + b_4 F_b \,. \tag{30}$$

As a result of numerical optimization we obtained the following coefficients of expansion:

$$b_1 = 1.35\,; \quad b_2 = 3\,; \quad b_3 = -\,0.06\,; \quad b_4 = 5.6\,. \tag{31}$$

Figure 7 compares the data of direct numerical modeling with the results of calculations by formula (16).

Correlation $\Pi_{(\varepsilon)ij}$. Since the correlation $\Pi_{(\varepsilon)}$ is under 0.001 at all points across the boundary layer (Fig. 2b) it is appropriate to take

$$\Pi_\varepsilon = 0\,. \tag{32}$$

To determine $\Pi_{(\varepsilon)ij}$, we can use approximation (11) or (21), which contain a number of the unknown coefficients. The latter can be determined from the data for the distributions of ε_{ij}.

Correlation $T_{(\varepsilon)ij}$. The approximation for this correlation is written in (10). For determination of T_ε, there is expression (17) and a simpler relation written in (26):

$$\frac{K}{\varepsilon^2} T_\varepsilon = C_\varepsilon \frac{K}{\varepsilon^2} \left[\frac{K}{\varepsilon} (R_{km} \varepsilon_{,m}) \right]_{,k} \,. \tag{33}$$

Having compared the data of DNS with the results of calculations by the above formula, the authors of [1] found that formula (33) describes satisfactorily the available data. Furthermore, this correlation is small as compared to the remaining terms of Eq. (3). Therefore we can recommend relation (33) for using in numerical calculations.

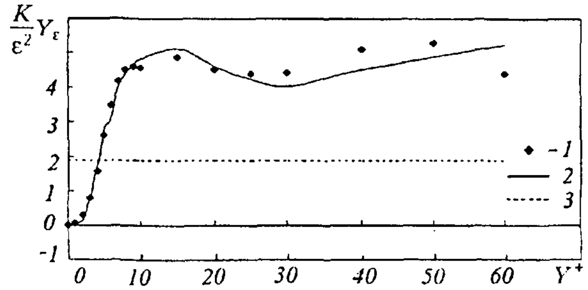


Fig. 7. Comparison of the results of calculations with the data of DNS for the correlation Y_ε: 1) data of DNS; 2) calculation by formula (16); 3) by the model of [2].

Correlation $D_{(\varepsilon)ij}$. There is no need to model this correlation within the framework of the complete second-order turbulence model since, it does not contain any unknown correlations. For calculations, we can use the relations written in (2) and (3).

Thus, we obtained approximations for all the correlations involved in the equation for the rate of dissipation of kinetic turbulence energy. As a result of modeling, we have a closed equation for the dissipation rate:

$$\frac{D\varepsilon}{Dt} = F_\varepsilon + P_{\varepsilon 1} + P_{\varepsilon 2} + P_{\varepsilon 3} + P_{\varepsilon 4} + \Pi_\varepsilon + T_\varepsilon + D_\varepsilon - Y_\varepsilon , \tag{34}$$

$$F_\varepsilon = -\frac{2\beta\nu}{\nu + a} g_i \varepsilon_{(\tau)i} , \quad P_{\varepsilon 1} = -2\varepsilon_{ik} U_{k,i} , \quad P_{\varepsilon 2} = \frac{P_k \varepsilon}{K} , \quad P_{\varepsilon 3} = 0 , \quad \Pi_\varepsilon = 0 ,$$

$$P_{\varepsilon 4} = 3\gamma_2 \frac{\varepsilon^2}{K} , \quad Y_\varepsilon = 3\gamma_4 \frac{\varepsilon^2}{K} , \quad D_\varepsilon = \nu \varepsilon_{,kk} , \quad T_\varepsilon = C_\varepsilon \left[\frac{K}{\varepsilon} (R_{km} \varepsilon_{,m}) \right]_{,k} ,$$

where $3\gamma_2$ and $3\gamma_4$ are determined from relations (28)-(31) while $C_\varepsilon = 0.15$.

The data of Fig. 4-7 show that Eq. (34) can be used in the entire flow region. And where the velocity gradients and the anisotropy are small the terms of the equation $P_{\varepsilon 3}$, Π_ε, T_ε, and D_ε are negligibly small. In the vicinity of the wall and in flows with a high degree of turbulence anisotropy, these terms yield a pronounced contribution to the total balance of the equation for the dissipation rate.

The analysis of relation (34) simultaneously with the approximations shows that in flows with a low anisotropy of turbulence the balance of the equation for the dissipation rate will be determined by the terms $P_{\varepsilon 4}$ and Y_ε, which, in turn, depend on the generation of turbulence energy, the normalized velocity gradient, the generation of the dissipation rate, and the degree of turbulence anisotropy. In the process of modeling of the equation for the dissipation rate ε, we obtained some data on the form of the approximations for correlations involved in the equation for the tensor dissipation function ε_{ij}. Furthermore, we determined the coefficients of the approximations mentioned. The coefficients that remain unknown must be found as a result of a more detailed investigation of the equation for ε_{ij}.

Thus, based on the data of direct numerical modeling [1] we obtained approximations for all the correlations involved in the equation for the rate of dissipation of kinetic turbulence energy. Earlier in numerical calculations use was made of an equation for the dissipation rate that is based on intuitive notions. Comparison of the results of calculating the individual terms of this equation with the data of DNS showed that the approximations for the unknown correlations are in poor agreement with the data of direct numerical modeling. This shows that the known equation for the dissipation rate cannot be recognized as satisfactory. Unlike it, in this work we modeled each correlation involved in the exact equation for the dissipation rate. The approximations obtained describe with a high degree of accuracy the data of DNS for developed flow in a channel. It can be assumed that the proposed equation will be more exact and, possibly, more universal. To confirm this, we need to test the proposed equation with reliable experimental results for flows of different types.

NOTATION

$R_{ij} = \langle u_i u_j \rangle$, single-point correlation of velocity pulsations; $F_{ij} = \langle u_i f_j \rangle)$ and $F_{(\tau)i} = \langle \tau f_i \rangle$, terms of generation due to the action of the external force; $P_{ij} = -(R_{ik}U_{kj} + R_{jk}U_{k,i})$, term of the generation of Reynolds stresses by the average velocity gradient; $\Phi_{ij} = (\langle \rho u_{ij} \rangle + \langle \rho u_{j,i} \rangle)/\rho$, term that contains the correlations of the rates of deformation with pressure pulsations; $\varepsilon_{ij} = \nu \langle u_{i,k} u_{k,j} \rangle$, tensor of dissipation rate; $D_{ij} = -[\langle u_i u_j u_k \rangle + (\langle p u_i \rangle \delta_{jk} + \langle p u_j \rangle \delta_{ik}) - \nu \langle u_i u_j \rangle_{,k}]_{,k}$, diffusion term; u_i, f_i, and p, pulsations of the velocity, the external force, and the pressure; ρ, density; ν, kinematic viscosity; δ_{ij}, Kronecker symbol, the angle brackets denote averaging, the comma preceding the subscript denotes differentiation; $\frac{D}{Dt} = \frac{\partial}{\partial t} + U_k \frac{\partial}{\partial x_k}$, U_k, averaged velocity; t, time; the correlations involved in the equation for the tensor of dissipation rate are: $F_{(\varepsilon)ij} = \nu(\langle u_{i,k} f_{j,k} \rangle + \langle u_{j,k} f_{i,k} \rangle)/\rho$, $P_{(\varepsilon 1)ij} = -(\varepsilon_{ik} U_{j,k} + \varepsilon_{jk} U_{i,k})$, $P_{(\varepsilon 2)ij} = -\nu U_{m,k}(\langle u_{i,k} u_{j,m} \rangle + \langle u_{j,k} u_{i,m} \rangle)$, $P_{(\varepsilon 4)ij} = -2\nu \langle u_{i,k} u_{j,m} u_{k,m} \rangle$, $P_{(\varepsilon 3)ij} = -\nu(\langle u_{i,k} u_m \rangle U_{j,km} + \langle u_{j,k} u_m \rangle U_{i,km})$, $Y_{(\varepsilon)ij} = -2\nu^2 \langle u_{i,km} u_{j,km} \rangle$, $\Pi_{(\varepsilon)ij} = -\frac{\nu}{\rho}(\langle u_{j,k} p_{,ik} \rangle + \langle u_{i,k} p_{,jk} \rangle)$, $D_{(\varepsilon)ij} = \nu \varepsilon_{ij,kk}$, $T_{(\varepsilon)ij} = -\nu \langle u_{i,k} u_{j,m} u_{k,m} \rangle_{,k}$; β, coefficient of volumetric expansion; g_i, free fall acceleration; $\varepsilon_{(\tau)i}$, dissipation term in the equation for the density of a turbulent heat flux; τ, temperature pulsation; $K = R_{ii}/2$, kinetic turbulence energy; $\varepsilon = \varepsilon_{ii}$, rate of dissipation of turbulence kinetic energy; $b_{ij} = \frac{R_{ij}}{2K} - \frac{1}{3}\delta_{ij}$, tensor of anisotropy of Reynolds stresses; $d_{ij} = \frac{\varepsilon_{ij}}{\varepsilon} - \frac{1}{3}\delta_{ij}$, tensor of anisotropy of dissipation processes; $P_k = -R_{mn}U_{m,n}$, generation of kinetic turbulence energy; $Y^+ = y u_\tau/\nu$, dimensionless distance from the wall; $u_\tau = (\nu U_{,y\mathrm{wall}})^{1/2}$, $U_{,y\mathrm{wall}}$, gradient of the averaged velocity on the wall; II_b, III_b, and F_b, scalar invariants that determine the degree of anisotropy of pulsation motion: $II_b = -b_{ik}b_{ki,}/2$, $III_b = b_{ik}b_{km}b_{mi}/3$, $F_b = 1 + 9II_b + 27III_b$, $b_{ij}^2 = b_{ik}b_{kj}$, $b_{ij}^3 = b_{ik}b_{km}b_{mj}$; II_d, III_d, and F_b, scalar invariants that determine the degree of anisotropy of dissipation processes: $II_d = -d_{ik}d_{ki,}/2$, $III_d = d_{ik}d_{km}d_{mi}/3$, $F_d = 1 + 9II_d + 27III_d$; the subscripts b and d indicate the method for determining scalar invariants; $X_1 = \frac{K}{\varepsilon}(S_{ij}S_{ij})^{1/2}$, normalized velocity gradient, $S_{ij} = \frac{1}{2}(U_{i,j} + U_{j,i})$; a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4, γ_1, γ_2, γ_3, γ_4, γ_5, C_{s2}, $a_{1\varepsilon}$, $a_{2\varepsilon}$, $a_{3\varepsilon}$, $C_{\varepsilon 1}$, $C_{\varepsilon 2}$, and C_ε, constants of the model.

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